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INDUCTION HEATING OF FLUIDS

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We consider problems of mathematical simulation of thermal processes during induction heating of fluids under conditions of conjugate heat exchange between the fluid flow and the channel walls of the induction heater, within which internal heat sources operate due to an induced electric current.

During heating of fluids in an induction flow-through heater, heat exchange occurs between the fluid flow and a solid body (the walls of the heater) with heat sources distributed in it. This is responsible for the variability of the surface temperature of the heater walls. This kind of problem has not as yet been studied adequately both theoretically and experimentally. Originally in works devoted to this problem the surface temperature was prescribed as a certain function of the coordinates. But this temperature distribution on the surface cannot be assigned a priori. It should be obtained as a result of simultaneous solution of equations for the fluid flow and the solid body, i.e., a conjugate problem of heat exchange [1]. Thus, mutual thermal effects of the solid body and the fluid, their thermophysical properties, and the distribution of internal heat sources are taken into account.

Solutions of conjugate problems of heat exchange for bodies of the simplest geometric form, predominantly for a laminar fluid flow, are given in [1]. It should be noted that even in this cases the form of the problems and their solutions is extremely complicated.

A criterion that would allow one to decide whether or not the problem belongs to the category of conjugate problems is the Brun number (the conjugation criterion):

$$\mathrm{Br} = \frac{\lambda_f b}{\lambda_s l} \mathrm{Pr}^m \mathrm{Re}^n . \tag{1}$$

The Brun number characterizes the relationship between the temperature drops over the wall thickness and in the boundary layer of the fluid flow:

$$\Delta\theta = A\varepsilon \frac{\lambda_f b}{\lambda_s l} \mathrm{Pr}^m \mathrm{Re}^n . \tag{2}$$

In this case it is assumed that if the Brun number is small enough, then the temperature drop over the wall thickness can be neglected, and the problem can be solved by a traditional method with account for boundary conditions on the external side of the wall. The critical (minimum) value of the Brun number ($\mathrm{Br}_{\min}$) can be determined from the condition

$$\Delta\theta = 0.05 . \tag{3}$$

By their thermophysical properties fluids can be divided into the following groups:

1. Elastic fluids (gases, vapors). They are distinguished by a low thermal conductivity coefficient (0.02–0.05 W/(m·K)) and a comparatively low Prandtl number ($\mathrm{Pr} \leq 1$), depending little on temperature [2-4].

2. Dropping liquids that are characterized by higher values of the thermal conductivity coefficient (0.15–0.7 W/(m·K)) and high values of the Prandtl number ($10-10^4$), strongly depending on temperature [2-4].

3. Liquid metals characterized by small values of the Prandtl number but high values of the thermal conductivity coefficient ($\mathrm{Pr} = (0.8-5)\cdot 10^{-2}$, $\lambda_f = 15-20$ W/(m·K)) [4].

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When determining the Brun number, it is necessary, apart from the thermophysical properties of the fluid, to assign the Reynolds number, which determines the regime of fluid motion, the thermal conductivity coefficient of the wall material, and the geometric ratio b/l.

If we take stainless steel [5] (λ_s = 15–20 W/(m·K)), i.e., an electrically conducting material used in contact with corrosive fluids in chemical technology and with food media, as the material of the wall and the geometric ratio b/l to be equal to the mean ratio between the wall thickness and diameter for standard tubes ($b/l \approx 0.17$, [6]), then the ratios of the Brun number in the turbulent regime of flow ($A = 0.021$; $m = 0.43$; $n = 0.8$; Re = 11000) to its minimum value for the groups of fluids enumerated above are:

1) $\mathrm{Br}/\mathrm{Br}_{\min} = 0.2 \div 1.5$,

2) $\mathrm{Br}/\mathrm{Br}_{\min} = 6 \div 115$,

3) $\mathrm{Br}/\mathrm{Br}_{\min} = 10 \div 90$.

In a laminar flow regime ($A = 0.66$; $m = 0.33$; $n = 0.5$; Re = 1000) the ratio $\mathrm{Br}/\mathrm{Br}_{\min}$ is much larger than unity and approaches five only in individual cases for substances of the second and third group.

Thus, heat exchange problems for substances of the second and third group (dropping liquids and liquid metals) should be considered as conjugate problems, at least for a turbulent flow regime. For substances of the second group (dropping liquids) this primarily refers to high-viscosity fluids distinguished by the highest values of the Prandtl number (oils, glycerin at low temperatures).

Considering problems of heat exchange in induction heating of substances of the third group (liquid metals), in addition to what has been said, it is necessary to take into account the ability of these substances to conduct an electric current, resulting in the appearance of heat sources directly in the fluid flow and body (electric) forces acting in the fluid.

The conjugation criterion introduced (the Brun number) characterizes the conditions of heat exchange in the direction perpendicular to the surface of the solid body interacting with the fluid. So, if the Brun number is small enough, then the temperature drop over the wall thickness is negligibly small and the temperature of the wall surface interacting with the fluid flow can be taken to be equal to the temperature of the outer wall surface, usually prescribed in the form of a certain law known a priori. However, in some cases the temperature distribution on the outer wall surface is not known in advance. Thus, even at small values of the Brun number the conditions at the boundary of interaction of the fluid flow with the wall are unknown and can be determined largely by the thermal conductivity of the wall in the longitudinal direction.

The problem of accounting for the thermal conductivity of the wall in the longitudinal direction can be solved if we consider the following particular cases.

1. Heat conduction along the wall is so large that the tempertature drop along the wall is negligibly small (much smaller than the temperature drop between the wall surface and the fluid heated). In this case the temperature along the wall can be considered constant.

It is possible to express the temperature drop between the wall surface and the fluid flow from the relation

$$P\eta = \alpha \Delta t_f S .$$

Assuming that a considerable portion of the heat released in the wall is redistributed in the form of a heat flux along it, we can calculate the temperature drop along the wall:

$$P\eta = \lambda_s \frac{\Delta t_s^{\mathrm{long}}}{L} \Pi b r \varepsilon^{\mathrm{long}} .$$

Then, after rearrangement we have

$$\Delta\theta = \frac{\Delta t_s^{\mathrm{long}}}{\Delta t_f} = \varepsilon^{\mathrm{long}} \frac{\lambda_f}{\lambda_s} \frac{b}{l} \left(\frac{L}{b}\right)^2 \mathrm{Nu} . \tag{4}$$

The quantity

$$\mathrm{Br}^{\mathrm{long}} = \frac{\lambda_f}{\lambda_s}\,\frac{b}{l}\left(\frac{L}{b}\right)^2 \mathrm{Pr}^m \mathrm{Re}^n \,, \tag{5}$$

obtained from Eq. (4), can be called the longitudinal Brun number. The critical (minimum) value of $\mathrm{Br}^{\mathrm{long}}$ can be found from the condition

$$\Delta\theta^{\mathrm{long}} \le 0.005 \,. \tag{6}$$

In this case if the value of $\mathrm{Br}^{\mathrm{long}}$ is smaller than or equal to the critical value, then the temperature of the wall in the longitudinal direction can be considered constant. If condition (6) is not fulfilled, the problem must be treated as a conjugate one even when $\mathrm{Br} \le \mathrm{Br}_{\mathrm{min}}$.

2. The longitudinal heat flux in the wall of the heater is negligibly small in comparison with the transverse heat flux.

The relationship between the longitudinal and transverse fluxes can be expressed in the following manner:

$$\frac{\Phi^{\mathrm{long}}}{\Phi} = \frac{\lambda_s \dfrac{\Delta t_s^{\mathrm{long}}}{L}\,\Pi b}{\dfrac{P\eta}{r}} \,.$$

Assuming that $\Delta t_s = P\eta / rMc$, we obtain $\Phi^{\mathrm{long}}/\Phi = \lambda_s \Pi b/(McL)$.

If

$$\frac{\lambda_s \Pi b}{McL} << 1 \,, \tag{7}$$

then we can neglect the longitudinal heat flux and assume that for each elementary portion of the heat-evolving surface the heat flux transferred to the fluid is equal to the intensity of heat releases on the given portion. This simplifies substantially the mathematical formulation of the heat exchange problem in spite of the fact that the problem remains conjugate.

3. The most complicated case is that where neither condition (6) nor (7) is fulfilled. Under these conditions the problem should be considered conjugate and the longitudinal heat flux in the wall should be taken into account in its formulation.

Information on modeling heat exchange apparatuses with internal heat sources, which include induction heaters, is extremely scarce [7].

In inductive heating, the fluid flow generally passes through a system of channels of an induction heater in whose walls there are internal heat sources produced by an induced electric flux. Structurally, the presence of adjacent channels is rational, since in this case both the inner and outer sides of the wall that divides the adjacent channels are a heat exchange surface of the induction heater.

We can present a mathematical description of heat exchange in such a heater in the form of a system of equations that involves: equations of convective heat exchange in the fluid flow in each channel; equations of heat conduction in the walls dividing the flows; conditions of thermal interaction between the fluid flows and the walls. It is evident that such a system of equations is very complex, and therefore it is necessary to seek ways of simplifying the mathematical description.

In the first approximation we assume that the temperature drop over the wall thickness of the heater is negligibly small, and the transverse heat flux in the wall is incommensurably larger than the longitudinal heat flux, i.e., the latter can be neglected (conditions (6) and (7) are satisfied).

Figure 1 presents schematically two adjacent channels of the induction heater. A system of equations of heat exchange in these channels, obtained with account for the assumptions made, is given below:

$$Mcdt' = q_I \Pi_I \mathrm{d}x + q'_{II} \Pi'_{II}\, \mathrm{d}x \,, \tag{8}$$

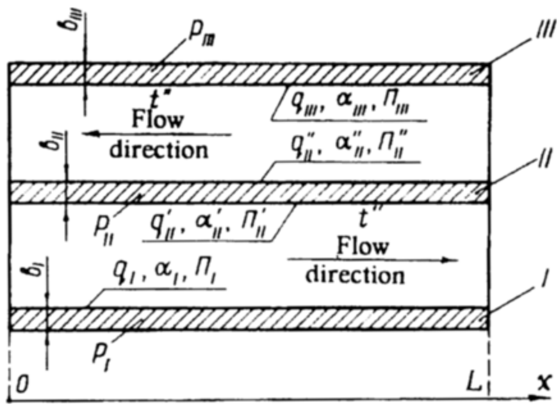

Fig. 1. Schematic of two adjacent channels of an induction heater: I) inner wall; II) wall dividing the adjacent channels; III) outer wall. x, coordinate in the flow direction (m).

$$Mcdt'' = -q''_{II}\Pi''_{II}dx - q_{III}\Pi_{III}dx, \tag{9}$$

$$q_I = \alpha_I (t_I - t'k'_p), \tag{10}$$

$$q'_{II} = \alpha'_{II} (t'_{II} - t'k'_p), \tag{11}$$

$$q_{III} = \alpha_{III} (t_{III} - t''k''_p), \tag{12}$$

$$q''_{II} = \alpha''_{II} (t''_{II} - t''k''_p), \tag{13}$$

$$q'_{II}\Pi'_{II} + q''_{II}\Pi''_{II} = p_{II}\Pi_{II,\mathrm{m}}b_{II}, \tag{14}$$

$$q_I = p_I b_I, \tag{15}$$

$$q_{III} = p_{III}\, b_{III}. \tag{16}$$

In essence, this system describes heat exchange between two conjugate fluxes at whose boundary of interaction distributed heat sources operate. Thus, the physical content of the problem differs somewhat from the actual one, i.e., there is no description of heat exchange in the walls. However, the given simplifications correspond to the adopted assumptions and allow us to obtain an analytically tractable mathematical model of heat exchange for two adjacent channels of an induction heater.

With Π, α, k_p, M, c, q_I, q_{III}, P_{II}, and b being constant, under the condition $k'_p = k''_p = k_p$, after transformation and nondimensionalization the system of equations (8)-(16) is reduced to a system of two equations:

$$\frac{d^2\theta'}{dX^2} = \frac{k_p}{F_1}(F_2 + F_3 + F_4), \tag{17}$$

$$\theta'' = \frac{F_1}{k_p}\frac{d\theta'}{dX} + \theta' - \frac{F_1F_2 + F_3F_5}{k_p}, \tag{18}$$

where

$$F_1 = \frac{Mc}{L}\left(\frac{1}{\Pi'_{II}\alpha'_{II}} + \frac{1}{\Pi''_{II}\alpha''_{II}}\right); \quad F_2 = \frac{\Pi_I q_I L}{Mc\Delta t};$$

$$F_3 = \frac{\Pi_{II,\mathrm{m}} b_{II} p_{II} L}{Mc\Delta t}; \quad F_4 = \frac{\Pi_{III} q_{III} L}{Mc\Delta t}; \quad F_5 = \frac{Mc}{\Pi''_{II}\alpha''_{II} L};$$

$$\theta' = \frac{t' - t_{\mathrm{in}}}{\Delta t}; \quad \theta'' - \frac{t'' - t_{\mathrm{in}}}{\Delta t}; \quad \Delta t = t_{\mathrm{fin}} - t_{\mathrm{in}}; \quad X = \frac{x}{L}.$$

The solutions of system (17)-(18) have the following form:

$$\theta' = -\frac{k_p}{F_1}(F_2 + F_3 + F_4)X^2 + E_1 X + E_2, \tag{19}$$

$$\theta'' = -\frac{k_p}{F_1}(F_2 + F_3 + F_4)X^2 + (E_1 - F_2 - F_3 - F_4)X + \frac{F_1}{k_p}E_1 + E_2 - \frac{F_1F_2 + F_3F_5}{k_p}, \tag{20}$$

where E_1 and E_2 are integration constants.

Using the notation

$$\frac{k_p}{F_1} = H_1; \quad F_2 + F_3 + F_4 = H_2; \quad \frac{F_1F_2 + F_3F_5}{k_p} = H_3,$$

we can represent Eqs. (19)-(20) more compactly in the form

$$\theta' = -H_1H_2X^2 + E_1X + E_2, \tag{21}$$

$$\theta'' = -\frac{H_1H_2}{2}X^2 + (E_1 - H_2)X + \frac{E_1}{H_2} + E_2 - H_3. \tag{22}$$

Two adjacent channels are just an element of the induction heat-exchange apparatus. Generally, several such elements can be present. When combining solutions for a pair of channels into a general solution for an arbitrary number of channels of an induction heater, it is necessary to construct a system of equations that represents the conditions that determine the order in which the fluid flow passes through the channels.

As an example, we can consider a twelve-channel cylindrical induction heater that was subjected to tests (see Fig. 2). The order in which the fluid flow traverses the channels of the heater is indicated by the figures: $1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 9 \to 10 \to 11 \to 12$.

In this case six pairs of adjacent channels are formed: 1, 12; 2, 11; 3, 10; 4, 9; 5, 8; 6, 7. Moreover, when the liquid flow leaves the preceding channel and enters the succeeding one, it reverses the direction of its motion. It is evident that the temperature of the fluid at the exit from the preceding channel should be equal to that at the entry into the succeeding one. The combination of these conditions has the following form:

$$\begin{gathered} \theta_{1,X=0} = 0; \ \theta_{1,X=1} = \theta_{2,X=0}; \ \theta_{2,X=1} = \theta_{3,X=0}; \\ \theta_{3,X=1} = \theta_{4,X=0}; \ \theta_{4,X=1} = \theta_{5,X=0}; \ \theta_{5,X=1} = \theta_{6,X=0}; \\ \theta_{6,X=1} = \theta_{7,X=1}; \ \theta_{7,X=0} = \theta_{8,X=1}; \ \theta_{8,X=0} = \theta_{9,X=1}; \\ \theta_{9,X=0} = \theta_{10,X=1}; \ \theta_{10,X=0} = \theta_{11,X=1}; \ \theta_{11,X=0} = \theta_{12,X=1}. \end{gathered} \tag{23}$$

The beginning of each inner channel (i.e., 1-6) is determined by the coordinate $X = 0$ in accordance with Fig. 1. Similar conditions can be compiled for an arbitrary number of channels.

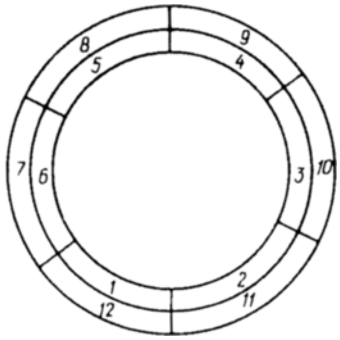

Fig. 2. Schematic of a twelve-channel cylindrical induction heater: 1-12) numbers of the channels.

Simultaneous solution of conditions (23) makes it possible to find integration constants in expressions (21)-(22) and thereby determine the dependence of the fluid flow temperature on the coordinate in an arbitrary channel of the heater. The distribution of temperatures on the heat-exchange surfaces of the heater can easily be found by using Eqs. (10)-(13) simultaneously with (21)-(22).

Thus, we have formulated and solved analytically the problem of induction heating of a fluid flow in a heater with an arbitrary number of adjacent channels under conditions of conjugate heat exchange between the fluid flow and the channel walls. The results of the solution can form a basis for rational design of induction heaters for fluids and give insight into the kinetics of fluid heating in induction heaters, i.e., heat exchange apparatuses with internal heat sources.

NOTATION

Br, Brun number; λ, thermal conductivity coefficient; b, wall thickness; l, characteristic dimension (equivalent diameter of the channel); Re, Pr, Reynolds and Prandtl numbers; $\Delta\theta$, relative temperature drop; ε, correction factor for the distortion of the temperature profile in the wall (in the case of a linear temperature distribution $\varepsilon = 1$); A, m, n, coefficients in the similarity equation; P, heat power of the heater; η, thermal efficiency of the heater; α, heat-transfer coefficient; Δt, temperature drop; S, heat-exchange surface of the heater; L, length of a heater channel; Π, perimeter of a heater channel; Nu, Nusselt number; Φ, heat flux; r, number of channels in the heater; M, mass flow rate of the fluid; c, heat capacity of the fluid; t, temperature; x, coordinate in the flow direction; q, density of the heat flux through the wall surface; k_p, coefficient of the distribution of temperature over the channel cross section; p, specific volumetric power of the internal heat sources; θ, dimensionless temperature. Subscripts and superscripts: f, fluid; s, wall; long, longitudinal; ′, inner channel; ″, outer channel; *I*, *II*, *III*, numbers of the walls; m, mean; in, initial; fin, final.

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